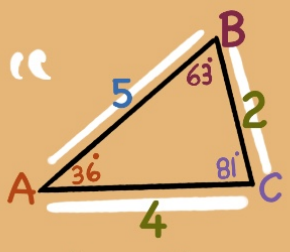
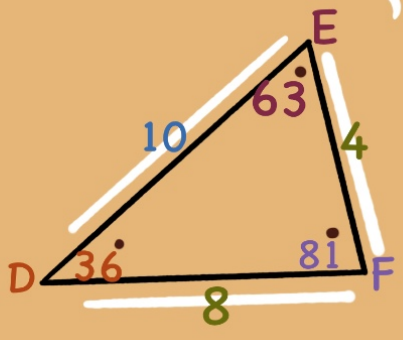
The Maths Society

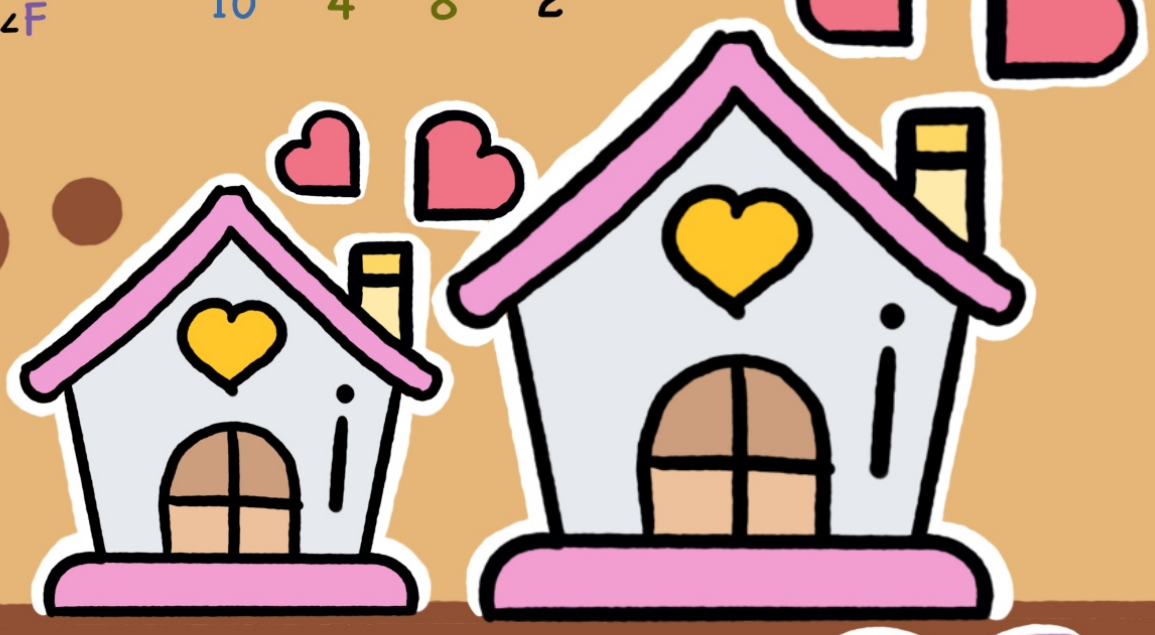
"Similarity"



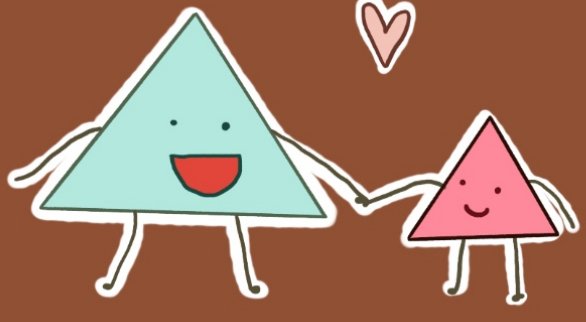


$\angle A \cong \angle D$
$\angle B \cong \angle E$
$\angle C \cong \angle F$

$$\frac{5}{10} = \frac{2}{4} = \frac{4}{8} = \frac{1}{2}$$

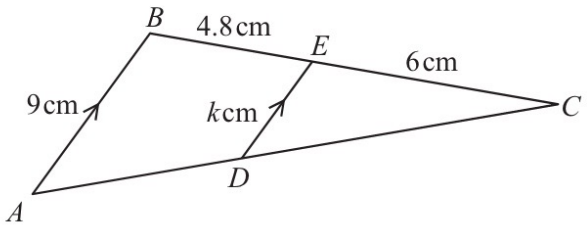


We're so similar!



Question 1

(a)



NOT TO SCALE

Triangles *CBA* and *CED* are similar.
AB is parallel to *DE*.
AB = 9 cm, *BE* = 4.8 cm, *EC* = 6 cm and *ED* = *k* cm.

[2]

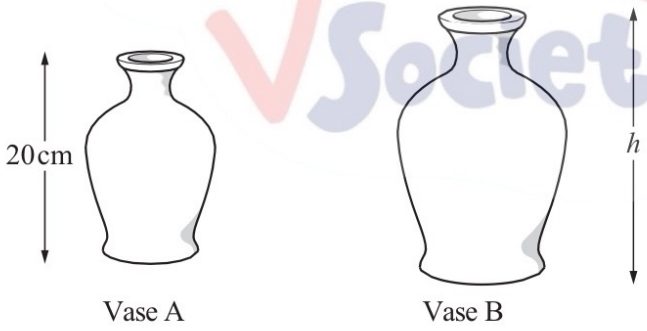
Work out the value of *k*.

$\frac{k}{9} = \frac{6}{6+4.8}$

$10.8k = 54$

$k = 5\text{ cm}$

(b)



NOT TO SCALE

The diagram shows two mathematically similar vases.
Vase A has height 20 cm and volume 1500 cm^3.
Vase B has volume 2592 cm^3.

Calculate *h*, the height of vase B.

[3]

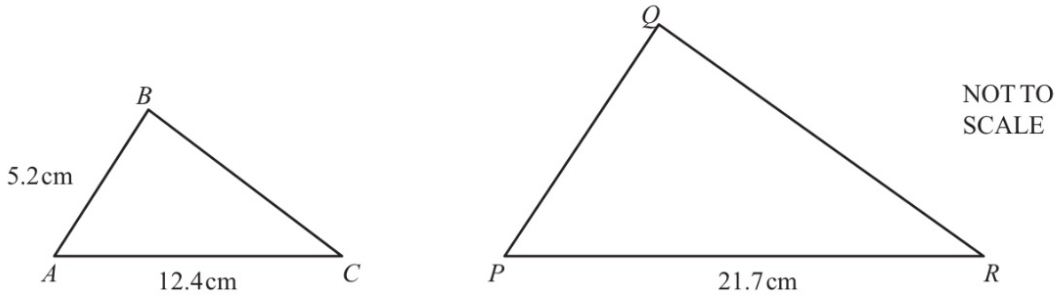
$\frac{1500}{2592} = \frac{(20)^3}{(k)^3}$

$\sqrt[3]{\frac{1500}{2592}} = \frac{20}{k}$

$k = 24\text{ cm}$

Question 2

Triangle ABC is similar to triangle PQR.

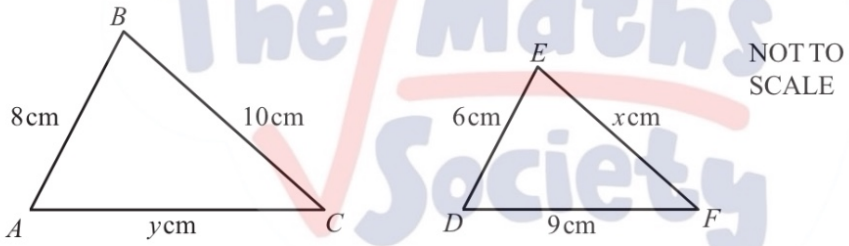


Find PQ.

$$\frac{PQ}{5.2} = \frac{21.7}{12.4}$$

$$PQ = 9.1\text{cm}$$

Question 3



Triangle ABC is similar to triangle DEF.

Calculate the value of

(a) x,

$$\frac{x}{10} = \frac{6}{8}$$

$$x = 7.5\text{cm}$$

[2]

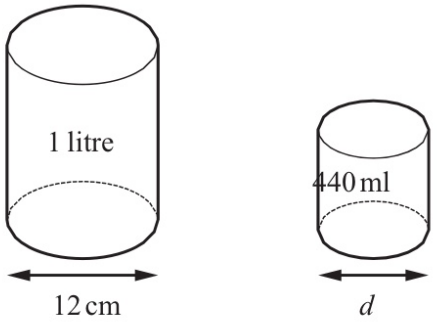
(b) y.

$$\frac{y}{9} = \frac{8}{6}$$

$$y = 12\text{ cm}$$

[2]

Question 4



NOT TO SCALE

Two cylindrical cans are mathematically similar.
The larger can has a capacity of 1 litre and the smaller can has a capacity of 440ml.

Calculate the diameter, d, of the 440ml can. [3]

$$\frac{d^3}{12^3} = \frac{440}{1000}$$

$$d = \sqrt[3]{\frac{440}{1000}} \times 12$$

$$= 9.13\text{cm}$$

Question 5

NOT TO SCALE

The two containers are mathematically similar in shape.
The larger container has a volume of 3456 cm^3 and a surface area of 1024 cm^2.
The smaller container has a volume of 1458 cm^3.

Calculate the surface area of the smaller container. [4]

$$\frac{(3456)^2}{(1458)^2} = \frac{(1024)^3}{(x)^3}$$

$$(1024)^3 \div \frac{4096}{729} = x^3$$

$$x = \sqrt[3]{1024^3 / \frac{4096}{729}}$$

$$x = 576\,cm^2$$

Question 6

The volumes of two similar cones are $36\pi\,cm^3$ and $288\pi\,cm^3$.
The base radius of the smaller cone is 3 cm.

Calculate the base radius of the larger cone. [3]

$$\frac{36\pi}{288\pi} = \frac{3^3}{x^3}$$

$$x^3 = 216$$

$$x = \sqrt[3]{216}$$

$$= 6\,cm$$

Question 7

A company sells cereals in boxes which measure 10 cm by 25 cm by 35 cm.

They make a special edition box which is mathematically similar to the original box.

The volume of the special edition box is $15\,120\,cm^3$.

Work out the dimensions of this box. [3]

vol of og $= 10 \times 25 \times 35$

$= 8750\,cm^3$

$$\frac{8750}{15120} = \frac{10^3}{x^3}$$

$$x^3 = 1728$$

$$x = 12\,cm$$

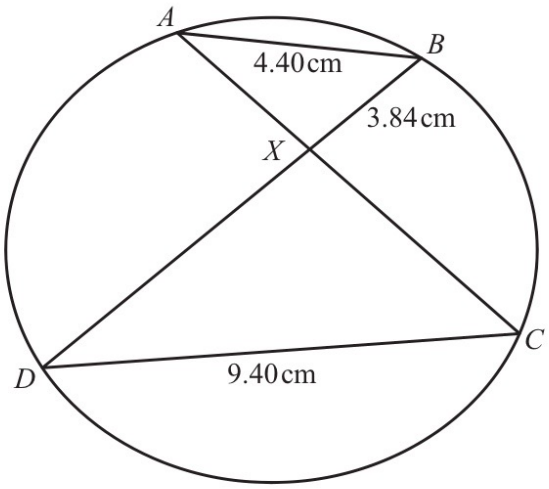
$$\frac{8750}{15120} = \frac{25^3}{x^3}$$

$$x = 30\,cm$$

$$\frac{8750}{15120} = \frac{35^3}{x^3}$$

$$x = 42\,cm$$

Question 1



NOT TO SCALE

A, B, C and D lie on a circle.
AC and BD intersect at X.

(a) Give a reason why angle BAX is equal to angle CDX. [1]

As the angles are in same segment.

(b) $AB = 4.40$ cm, $CD = 9.40$ cm and $BX = 3.84$ cm.

(i) Calculate the length of CX. [2]

$$\frac{CX}{3.84} = \frac{9.4}{4.4}$$

$$CX = 8.2 \text{ cm}$$

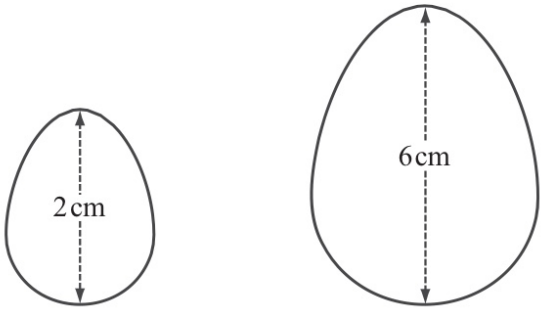
(ii) The area of triangle ABX is 5.41 cm^2.

Calculate the area of triangle CDX. [2]

$$\frac{x}{5.41} = \frac{(9.4)^2}{(4.4)^2}$$

$$x = 24.69 \text{ cm}^2$$

Question 2



NOT TO SCALE

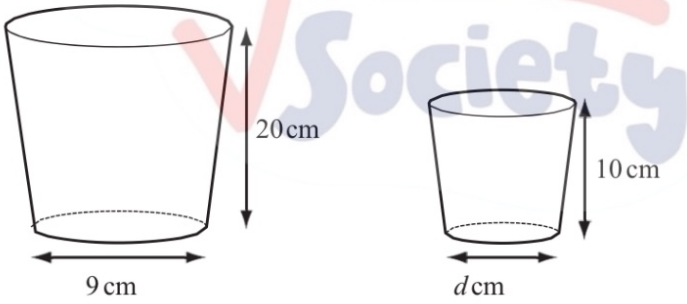
A company makes solid chocolate eggs and their shapes are mathematically similar.
The diagram shows eggs of height 2 cm and 6 cm.
The mass of the small egg is 4 g.

Calculate the mass of the large egg. [2]

$$\frac{2^3}{6^3} = \frac{4}{x}$$

$$x = 108g$$

Question 3



NOT TO SCALE

The diagrams show two mathematically similar containers.
The larger container has a base with diameter 9 cm and a height 20 cm.
The smaller container has a base with diameter d cm and a height 10 cm.

(a) Find the value of d. [1]

$$\frac{d}{9} = \frac{10}{20}$$

$$d = 4.5cm$$

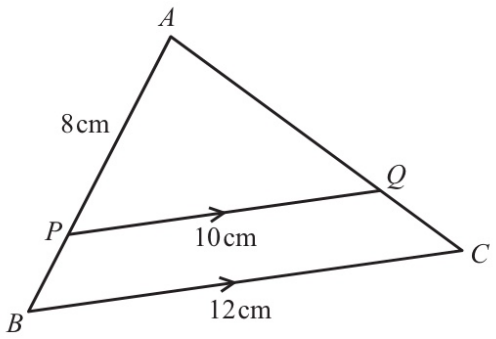
(b) The larger container has a capacity of 1600 ml. [2]

Calculate the capacity of the smaller container.

$$\frac{(10)^3}{(20)^3} = \frac{x}{1600}$$

$$x = 200ml$$

Question 4



NOT TO SCALE

APB and *AQC* are straight lines. *PQ* is parallel to *BC*.
AP = 8 cm, *PQ* = 10 cm and *BC* = 12 cm. [2]
Calculate the length of *AB*.

$\frac{AB}{8} = \frac{12}{10}$

$AB = 9.6\ cm$

Question 5

A car manufacturer sells a similar, scale model of one of its real cars.

(a) The fuel tank of the real car has a volume of 64 litres and the fuel tank of the model has a volume of 0.125 litres.
Show that the length of the real car is 8 times the length of the model car. [2]

$\sqrt[3]{64} = 4$

$\sqrt[3]{0.125} = 0.5$

$\frac{4}{0.5} = 8$ ←

(b) The area of the front window of the model is 0.0175 m^2.
Find the area of the front window of the real car. [2]

$8^2 = 64$

$0.0175 \times 64 = 1.12 m^2$

Question 6

A cylindrical glass has a radius of 3 centimetres and a height of 7 centimetres.
A large cylindrical jar full of water is a similar shape to the glass.
The glass can be filled with water from the jar exactly 216 times.
Work out the radius and height of the jar. [3]

$r = 3\text{ cm}$ $h = 7\text{cm}$ $V = \pi r^2 h$

height of big $= \frac{7}{8}r$

$\pi \times 3^2 \times 7 \times 216 = \pi r^2 \frac{7}{8} r$

$13680 = \frac{7}{3} r^3$

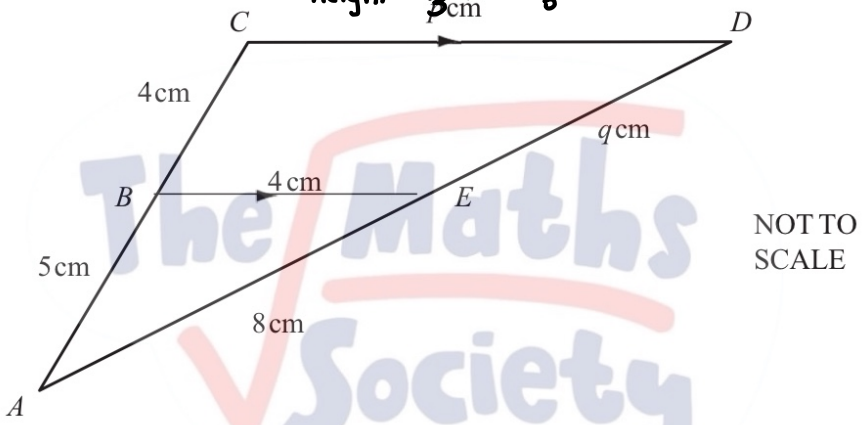
$r^3 = 5832$

$r = 18\text{ cm}$

height $= \frac{7}{8} r = \frac{7}{8} \times 18 = 42\text{cm}$

Question 7

(a)



In the diagram triangles ABE and ACD are similar.
BE is parallel to CD.
$AB = 5$ cm, $BC = 4$ cm, $BE = 4$ cm, $AE = 8$ cm, $CD = p$ cm and $DE = q$ cm.
Work out the values of p and q.

$\frac{AC}{AB} = \frac{CD}{BE}$

$\frac{9}{5} = \frac{p}{4}$

$p = 7.2\text{cm}$

$\frac{q}{5} = \frac{q+8}{8}$

$q = 6.4\text{cm}$

[4]

(b) A spherical balloon of radius 3 metres has a volume of 36π cubic metres.
It is further inflated until its radius is 12 m.
Calculate its new volume, leaving your answer in terms of π.

$\frac{(3)^3}{(12)^3} = \frac{36\pi}{x}$

$x = 2304\pi$

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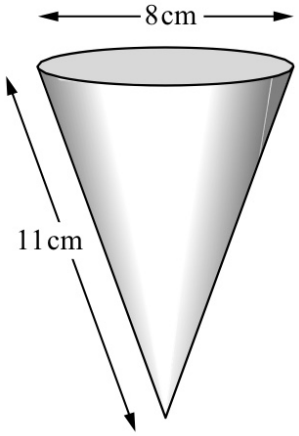
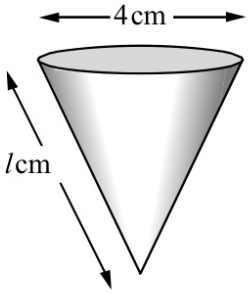
[2]

Question 8





NOT TO SCALE

The two cones are similar.

(a) Write down the value of l. [1]

$\frac{4}{8} = \frac{l}{11}$

$l = 5.5\text{cm}$

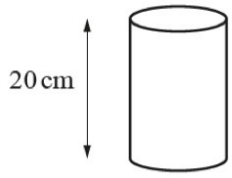
(b) When full, the larger cone contains 172 cm^3 of water.
How much water does the smaller cone contain when it is full? [2]

$\frac{172}{x} = \frac{(8)^3}{(4)^3}$

$x = 21.5\text{cm}^3$

Question 1

(a)



NOT TO SCALE

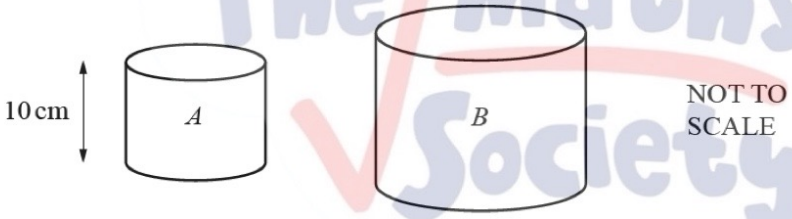
A cylinder has height 20cm.
The area of the circular cross section is 74cm^2.

Work out the volume of this cylinder. [1]

$V = 74 \times 20$

$= 1480\text{cm}^3$

(b) Cylinder *A* is mathematically similar to cylinder *B*.



NOT TO SCALE

The height of cylinder *A* is 10 cm and its surface area is $440\,\text{cm}^2$.
The surface area of cylinder *B* is $3960\,\text{cm}^2$.

Calculate the height of cylinder *B*. [3]

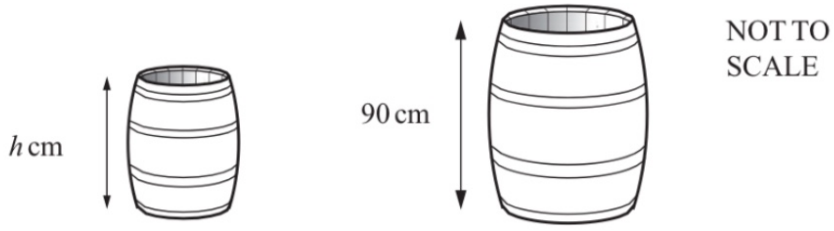
$\left(\frac{10}{x}\right)^2 = \frac{440}{3960}$

$x^2 = 900$

$x = 30\text{cm}$

Question 2

The two barrels in the diagram are mathematically similar.



The smaller barrel has a height of h cm and a capacity of 100 litres.
The larger barrel has a height of 90 cm and a capacity of 160 litres.

Work out the value of h. [3]

$$\frac{(h)^3}{(90)^3} = \frac{1000}{1600}$$

$$h^3 = 455\,625$$

$$h = 76.9 \text{ cm}$$

Question 3

Two bottles and their labels are mathematically similar.
The smaller bottle contains 0.512 litres of water and has a label with area 96 cm^2.
The larger bottle contains 1 litre of water.

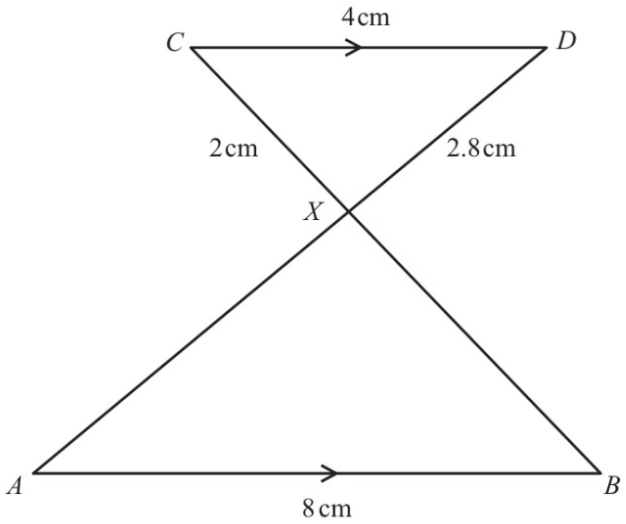
Calculate the area of the larger label. [3]

$$\frac{(0.512)^2}{(1)^2} = \frac{(96)^3}{x^3}$$

$$x^3 = 3375000$$

$$x = 150 \text{ cm}^2$$

Question 4



NOT TO SCALE

In the diagram, AB and CD are parallel.
AD and BC intersect at X.
$AB = 8$ cm, $CD = 4$ cm, $CX = 2$ cm and $DX = 2.8$ cm.

(a) Complete this mathematical statement. [1]

Triangle ABX is ..... similar ..... to triangle DCX.

(b) Calculate AX. [2]

$$\frac{AX}{2.8} = \frac{8}{4}$$

$$AX = 5.6\text{cm}$$

(c) The area of triangle ABX is y cm^2.

Find the area of triangle DCX in terms of y. [1]

$$\frac{y}{4}$$

Question 5

Two cups are mathematically similar.
The larger cup has capacity 0.5 litres and height 8 cm.
The smaller cup has capacity 0.25 litres.

Find the height of the smaller cup. [3]

$$\frac{500}{250} = \frac{(8)^3}{(x)^3}$$

$$x^3 = 256$$

$$x = 6.35\text{cm}$$

Question 6

The length of a backpack of capacity 30 litres is 53 cm.

Calculate the length of a mathematically similar backpack of capacity 20 litres. [3]

$$\frac{30\,000}{20\,000} = \frac{(53)^3}{x^3}$$

$$x^3 = \frac{297754}{3}$$

$$x = 46.3\text{cm}$$

Question 1

Two containers are mathematically similar.
Their volumes are 54 cm³ and 128 cm³.
The height of the smaller container is 4.5 cm.

Calculate the height of the larger container.

$$\frac{54}{128} = \frac{4.5^3}{x^3}$$

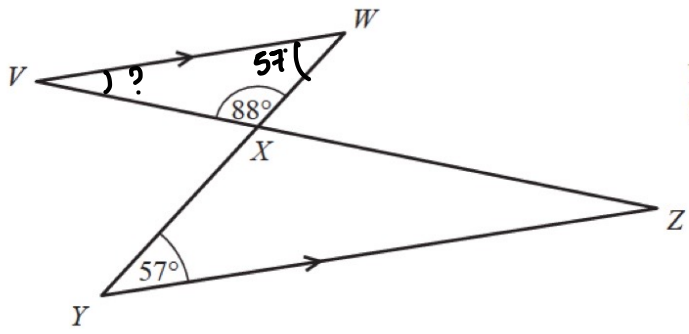
$$x^3 = 216$$

$$x = 6\text{cm}$$

[3]

Question 2

(a)



NOT TO SCALE

Two straight lines VZ and YW intersect at X.
VW is parallel to YZ, angle $XYZ = 57°$ and angle $VXW = 88°$.

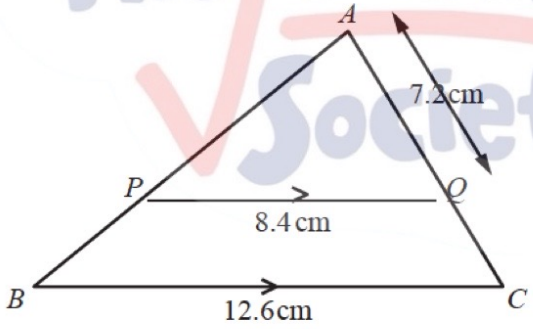
Find angle WVX.

$WVX = 180° - 88° - 57°$

$= 35°$

[2]

(b)



NOT TO SCALE

ABC is a triangle and PQ is parallel to BC.
$BC = 12.6$ cm, $PQ = 8.4$ cm and $AQ = 7.2$ cm.

Find AC.

$$\frac{AC}{7.2} = \frac{12.6}{8.4}$$

$AC = 10.8$cm

[2]

Question 3

A car, 4.4 metres long, has a fuel tank which holds 65 litres of fuel when full.
The fuel tank of a mathematically similar model of the car holds 0.05 litres of fuel when full.

Calculate the length of the model car in centimetres. [3]

$$\frac{65}{0.05} = \frac{4.4^3}{x^3}$$

$$x = 0.403 \text{ m}$$

$$= 40.3 \text{cm}$$

Question 4

Two similar vases have heights which are in the ratio 3 : 2.

(a) The volume of the larger vase is 1080 cm^3.
Calculate the volume of the smaller vase.

$$\frac{(3)^3}{(2)^3} = \frac{1080}{x}$$

$$x = 320 \text{ cm}^3$$

[2]

(b) The surface area of the smaller vase is 252 cm^2.
Calculate the surface area of the larger vase.

$$\frac{(3)^2}{(2)^2} = \frac{x}{252}$$

$$x = 567 \text{ cm}^2$$

[2]

Question 5

$h = 2m$ $\quad V = 5m^3$

A statue two metres high has a volume of five cubic metres.
A similar model of the statue has a height of four centimetres.

$h = 4cm = 0.04m$

(a) Calculate the volume of the model statue in cubic centimetres.

$$\frac{(200)^3}{(4)^3} = \frac{50^3}{x}$$

$$x = 40cm^3$$

[2]

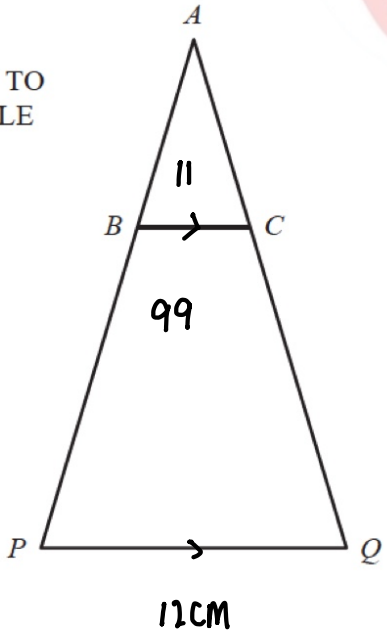
(b) Write your answer to **part (a)** in cubic metres.

$0.00004\ m^3$

[1]

Question 6

NOT TO SCALE



The area of triangle APQ is 99 cm^2 and the area of triangle ABC is 11 cm^2. BC is parallel to PQ and the length of PQ is 12 cm.

Calculate the length of BC.

[3]

$$\frac{(12)^2}{(BC)^2} = \frac{99}{11}$$

$$BC^2 = 16$$

$$BC = 4cm$$